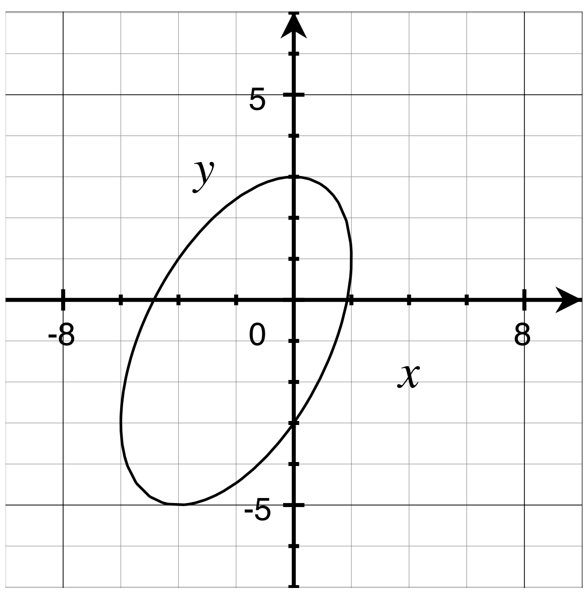
Implicit differentiation worksheet for Calculus 1

Determine dy/dx for each of the following.

(1) $y = x^2 + xy$
(2) $x^2y + y = 3$
(3) $x^{1/4} + y^{1/4} = 2$
(4) $x^{1/3} + y^{1/3} = 7$
(5) $\sqrt{x} + \sqrt{y} = 25$
(6) $x^2 + y^2 = 1.1$
(7) $x^3 + y^3 = \sqrt{5}$
(8) $x + \sin(y) = y + 1$
(9) $y\sqrt{x} + x\sqrt{y} = 16$
(10) $x^2 + xy - y^3 = xy^2$
(11) $x^2 + y^2 = \sqrt{7}$
(12) $x^{2/3} + y^{2/3} = a^{2/3}$ (a is a constant)
(13) $x^a y^2 + x^b y + x^c = 0$ (a, b, c constants)
(14) $\sin(xy) = 2x + 5$
(15) $x\ln(y) + y^3 = \ln(x)$
(16) $e^{\cos(y)} = x^3\sin(y)$

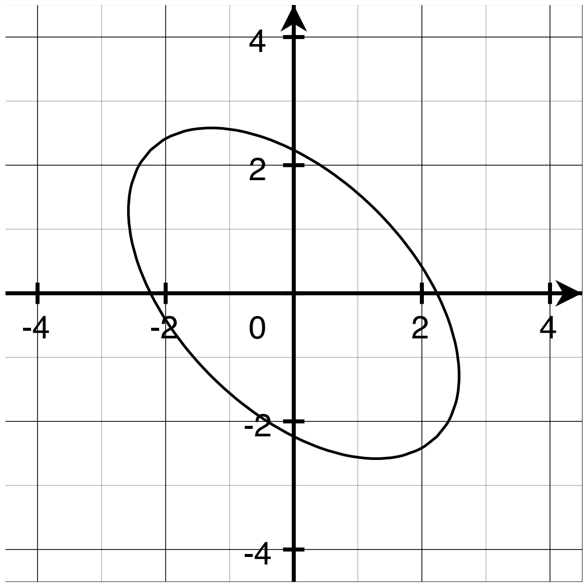
Determine d^2y/dx^2 for each of the following.

(17) $1 - xy = x - y^2$
(18) $x - y = (x + y)^2$
(19) $x^{2/3} + y^{2/3} = 8$
(20) $\sin(x) - 4\cos(y) = y$



For the curve $x^2 + y^2 - xy + 3x - 9 = 0$ (above),

(21) Determine dy/dx.
(22) Where do the horizontal tangent lines occur?
(23) Where do the vertical tangent lines occur ($dy/dx = \pm\infty$)?
(24) Determine d^2y/dx^2.



For the curve $x^2 + xy + y^2 = 5$ (above),

(25) Determine dy/dx.
(26) Where do the horizontal tangent lines occur?
(27) Where do the vertical tangent lines occur ($dy/dx = \pm\infty$)?
(28) Determine d^2y/dx^2.

Consider the equation

$$(\cos x)y^2 + (3\sin x - 1)y + (7x - 2) = 0$$

(29) Check that $x = 0$, $y = 2$ satisfies this equation.
(30) Find dy/dx at the point $(0, 2)$ using implicit differentiation.
(31) Use the quadratic formula to solve for y in terms of x. (Should you use "+" or "−"? Why?)
(32) Would you like to find dy/dx using that formula for y? (Me neither...)

Find $f'(x)$ in terms of $g(x)$ and $g'(x)$, where $g(x) > 0$ for all x. (Hint: if a is a constant then $g(a)$ is constant.)

(33) $f(x) = g(x)^3$
(34) $f(x) = g(x)(x - a)$
(35) $f(x) = g(a)(x - a)$
(36) $f(x) = g(x + g(x))$
(37) $f(x) = \dfrac{g(x)}{x - a}$
(38) $f(x) = \dfrac{1}{g(x)}$
(39) $f(x) = g(xg(a))$
(40) $f(x) = \sqrt{g(x)^2}$
(41) $f(x) = \sqrt{g(x^2)}$
(42) $f(2x + 3) = g(x^2)$